

NAG Toolbox for MATLAB

f08ta

1 Purpose

f08ta computes all the eigenvalues and, optionally, all the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$Az = \lambda Bz, \quad ABz = \lambda z \quad \text{or} \quad BAz = \lambda z,$$

where A and B are symmetric, stored in packed format, and B is also positive-definite.

2 Syntax

```
[ap, bp, w, z, info] = f08ta(itype, jobz, uplo, n, ap, bp)
```

3 Description

f08ta first performs a Cholesky factorization of the matrix B as $B = U^T U$, when **uplo** = 'U' or $B = LL^T$, when **uplo** = 'L'. The generalized problem is then reduced to a standard symmetric eigenvalue problem

$$Cx = \lambda x,$$

which is solved for the eigenvalues and, optionally, the eigenvectors; the eigenvectors are then backtransformed to give the eigenvectors of the original problem.

For the problem $Az = \lambda Bz$, the eigenvectors are normalized so that the matrix of eigenvectors, Z , satisfies

$$Z^T A Z = \Lambda \quad \text{and} \quad Z^T B Z = I,$$

where Λ is the diagonal matrix whose diagonal elements are the eigenvalues. For the problem $ABz = \lambda z$ we correspondingly have

$$Z^{-1} A Z^{-T} = \Lambda \quad \text{and} \quad Z^T B Z = I,$$

and for $BAz = \lambda z$ we have

$$Z^T A Z = \Lambda \quad \text{and} \quad Z^T B^{-1} Z = I.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **itype** – int32 scalar

Specifies the problem type to be solved.

itype = 1

$$Az = \lambda Bz.$$

itype = 2

$$ABz = \lambda z.$$

itype = 3

$$BAz = \lambda z.$$

2: **jobz** – string

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

3: **uplo** – string

If **uplo** = 'U', the upper triangles of A and B are stored.

If **uplo** = 'L', the lower triangles of A and B are stored.

Constraint: **uplo** = 'U' or 'L'.

4: **n** – int32 scalar

n , the order of the matrices A and B .

Constraint: $n \geq 0$.

5: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n symmetric matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

6: **bp**(*) – double array

Note: the dimension of the array **bp** must be at least $\max(1, n \times (n + 1)/2)$.

The upper or lower triangle of the symmetric matrix B , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of B must be stored with element B_{ij} in **bp**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of B must be stored with element B_{ij} in **bp**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work

5.4 Output Parameters

1: **ap(*) – double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

The contents of **ap** are destroyed.

2: **bp(*) – double array**

Note: the dimension of the array **bp** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

The triangular factor U or L from the Cholesky factorization $B = U^T U$ or $B = LL^T$, in the same storage format as B .

3: **w(*) – double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

If **info** = 0, the eigenvalues in ascending order.

4: **z(ldz,*) – double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** = 'V', **ldz** $\geq \max(1, \mathbf{n})$;
ldz ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'V', then if **info** = 0, **z** contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows:

if **itype** = 1 or 2, $Z^T B Z = I$;
 if **itype** = 3, $Z^T B^{-1} Z = I$.

If **jobz** = 'N', **z** is not referenced.

5: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **itype**, 2: **jobz**, 3: **uplo**, 4: **n**, 5: **ap**, 6: **bp**, 7: **w**, 8: **z**, 9: **ldz**, 10: **work**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

f07gd or f08ga returned an error code:

$\leq \mathbf{n}$ if **info** = i , f08ga failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero;

$> \mathbf{n}$ if **info** = $\mathbf{n} + i$, for $1 \leq i \leq \mathbf{n}$, then the leading minor of order i of B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

7 Accuracy

If B is ill-conditioned with respect to inversion, then the error bounds for the computed eigenvalues and vectors may be large, although when the diagonal elements of B differ widely in magnitude the eigenvalues and eigenvectors may be less sensitive than the condition of B would suggest. See Section 4.10 of Anderson *et al.* 1999 for details of the error bounds.

The example program below illustrates the computation of approximate error bounds.

8 Further Comments

The total number of floating-point operations is proportional to n^3 .

The complex analogue of this function is f08tn.

9 Example

```
itype = int32(1);
jobz = 'No vectors';
uplo = 'U';
n = int32(4);
ap = [0.24;
      0.39;
      -0.11;
      0.42;
      0.79;
      -0.25;
      -0.16;
      0.63;
      0.48;
      -0.03];
bp = [4.16;
      -3.12;
      5.03;
      0.56;
      -0.83;
      0.76;
      -0.1;
      1.09;
      0.34;
      1.18];
[apOut, bpOut, w, z, info] = f08ta(itype, jobz, uplo, n, ap, bp)

apOut =
    0.0875
    0.4683
    0.5244
    0.4892
   -0.6812
   -0.3775
   -0.4476
   -0.4576
   -0.9487
   -1.6875
bpOut =
    2.0396
   -1.5297
    1.6401
    0.2746
   -0.2500
    0.7887
   -0.0490
    0.6189
    0.6443
    0.6161
```

```
w =  
  -2.2254  
  -0.4548  
   0.1001  
   1.1270  
z =  
   0      0      0      0  
info =  
      0
```
